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Determining the Weak Phase γ in the Presence of Rescattering [1]

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Abstract

We suggest a new technique to determine the CKM phase γ *without* neglecting the (soft) final state rescattering effects. We use (time integrated) B meson decay rates to π's and K's. A set of 5 $\Delta S = 0$ (or 1 $\Delta S = 0$ and 4 $\Delta S = 1$) decay rates is used to compute the strong phases and magnitudes of the tree level and penguin contributions as functions of γ. These are used to *predict* a $\Delta S = 1$ ($\Delta S = 0$) $B_{d/s}$ decay rate as a function of γ (using $SU(3)$ symmetry). The measurement of this decay rate then gives γ. We illustrate this technique using different cases. Most of the decay modes we use are expected to be accessible at the B-factories (e^+e^- or hadron machines).

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1 Introduction

Determining the angles of the CKM unitarity triangle, denoted by α, β and γ, is one of the important aims of the B-factories. Methods have been suggested to determine γ ($\equiv$ Arg $(-V_{ub}^{\star}V_{ud}/V_{cb}^{\star}V_{cd})$ where V is the CKM matrix) using decays of B_d, B^+ and B_s (and their CP-conjugates) to two pseudoscalars belonging to the $SU(3)$ octet including the effects of the Electroweak Penguin (EWP) diagrams. These methods rely on the flavor $SU(3)$ symmetry.

Many of these methods neglect the effects of (soft) final state rescattering. In particular, the decay amplitude for $B^+ \to \pi^+ K^0$ is assumed to contain only the weak phase $e^{i\pi}$ from the penguin diagram with the top quark in the loop. Tree level operators have the weak phase $e^{i\gamma}$, but since they have the transition $\bar{b} \to \bar{s}u\bar{u}$, they contribute, in the absence of rescattering, only through annihilation to this decay. Annihilation contributions are argued to be small since they are suppressed by f_B/m_B (in the absence of significant rescattering effects). Assuming that the $B^+ \to \pi^+ K^0$ amplitude has no $e^{i\gamma}$ weak phase, references [1, 2, 3, 4] have suggested methods to determine γ.

However, rescattering from an intermediate state, for example $\pi^0 K^+$, created by a (color-allowed) spectator decay of a B^+ due to tree level operators, can generate a significant amplitude with the weak phase $e^{i\gamma}$ in the decay $B^+ \to \pi^+ K^0$ as well. So it would be better to have a method to determine γ which does not use the assumption of no rescattering effects. Rescattering might also enhance annihilation contributions [5] and thus necessitates their inclusion.

Buras and Fleischer [6] gave a method to determine γ *without* neglecting rescattering using $B_d \to \pi^- K^+$, $B^+ \to \pi^+ \pi^0$ decays and time *dependent* measurements of the $B_d \to \pi^0 K_S$ decay. For this method, they also require time dependent analysis of, for example, $B_d \to J/\psi K_S$ to measure β. Gronau and Pirjol [7] suggested a method using time independent measurements of *all* the $B_d \to \pi K$ and $B_s \to \pi K$ modes. In their method also rescattering effects are included. However, it might be diffcult to measure the neutral

Case	Modes used $\Delta S = 0$	Modes used $\Delta S = 1$
1	$B^+ \to \pi^+\pi^0,\ B_d \to \pi^+\pi^-, \pi^0\pi^0$ $\bar{B}_d \to \pi^+\pi^-, \pi^0\pi^0$	$B_d \to \pi^- K^+, \pi^0 K^0$ $B_s \to \pi^+\pi^-$ (or $\pi^0\pi^0$)
2	$B^+ \to \pi^+\pi^0,\ B_d \to \pi^+\pi^-, \pi^0\pi^0$ $\bar{B}_d \to \pi^+\pi^-, \pi^0\pi^0$	$B_s \to K^+K^-$ (CP-averaged)
3	$B^+ \to \pi^+\pi^0,\ B_d \to \pi^+\pi^-, \pi^0\pi^0$ $B_d \to K^+K^-$	$B_d \to \pi^0 K^0, \pi^- K^+$ $\bar{B}_d \to \pi^0 \bar{K}^0, \pi^+ K^-$
4	$B^+ \to \pi^+\pi^0$ $B_s \to \pi^+ K^-$ (or $\pi^0 \bar{K}^0$) (CP-averaged)	$B_d \to \pi^0 K^0, \pi^- K^+$ $\bar{B}_d \to \pi^0 \bar{K}^0, \pi^+ K^-$

Table 1: The 6 (or 8) B decay modes used by each of the 4 cases to determine γ.

modes of B_s decays since that will involve tagging at hadron machines.

In this paper, we suggest a technique to determine γ *including* rescattering effects (and the EWP operators) using B meson decays to π's and K's. We will illustrate this technique using four cases; see table 1.

We do *not* require any time dependent studies. The strategy is as follows. In cases 1 and 2, using 5 $\Delta S = 0$ decay modes, we determine the strong phases and magnitudes of the tree level and penguin contributions as functions of γ (assuming flavor $SU(2)$ symmetry). Then, using flavor $SU(3)$ symmetry, we *predict* the rate for *one* $\Delta S = 1$ mode in case 2. In case 1, two $\Delta S = 1$ modes have to be measured to make a prediction for a third $\Delta S = 1$ mode. The measurement of the decay for which we have a prediction (as a function of γ) then determines γ. A similar idea can be applied to predict a $\Delta S = 0$ decay mode as a function of γ using measurements of $\Delta S = 1$ (and some $\Delta S = 0$) modes (cases 3 and 4).

The $B^+ \to \pi^+\pi^0$, $B_d \to \pi\pi$, πK modes should be relatively accessible at the e^+e^- and hadron machines. For the B_d decays to a CP eigenstate, we require (external) tagging (*i.e.,* the CP-averaged decay rate is not sufficient). The B_s decay modes were accessible at LEP1 and will be accessible at hadron machines. The $B_s \to \pi\pi$ decay mode (case 1) might be hard to measure since

it requies tagging whereas in cases 2 and 4, the B_s modes are either "self-tagging" ($\pi^+ K^-$) or a CP-averaged measurement is sufficient (for $K^- K^+$, $\pi^0 \bar{K}^0$). In case 1, we show that if we measure additional B_d modes, a CP-averaged measurement of the decay rate $B_s \to \pi\pi$ is sufficient.

Although flavor $SU(3)$ symmetry is used in all the four cases (as in all the other methods mentioned above), in the last section, we discuss how to take into account $SU(3)$ breaking.

2 Cases 1 and 2

We will write the decay amplitudes for the decays $B_i \to M\ M$, where M is a pseudoscalar belonging to the flavor $SU(3)$ octet, in terms of the 5 linearly independent $SU(3)$ invariant amplitudes denoted by $C_3^{T,P}$, $C_6^{T,P}$, $C_{15}^{T,P}$, $A_3^{T,P}$ and $A_{15}^{T,P}$ (where T and P stand for the parts of these amplitudes generated by tree level and penguin operators, respectively). These $SU(3)$ invariant amplitudes include rescattering effects. The annihilation amplitudes, $A_{3,15}$, are the ones in which the quark index i of B_i is contracted directly with the Hamiltonian. Neglecting rescattering effects is equivalent to assuming $C_3^T - C_6^T - C_{15}^T = 0$ and $A_{3,15} \sim f_B/m_B$. For example, the tree level part of the decay amplitude $\mathcal{A}\,(B^+ \to \pi^+ K^0)$ contains this combination of the C^T amplitudes and A_{15}^T.

In this notation [8], the amplitudes for $B \to \pi\pi$ decays can be written as

$$
\begin{aligned}
-\sqrt{2}\mathcal{A}(B^+ \to \pi^+\pi^0) &= 8\ (\lambda_u^{(d)} C_{15}^T + \sum_q \lambda_q^{(d)} C_{15,q}^P) \\
&= -3\ I_2, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{2}\mathcal{A}(B_d \to \pi^0\pi^0) &= \lambda_u^{(d)}(C_3^T + C_6^T - 5C_{15}^T) + \sum_q \lambda_q^{(d)}(C_{3,q}^P + C_{6,q}^P - 5C_{15,q}^P) \\
&\quad + \lambda_u^{(d)}\left(2A_3^T + A_{15}^T\right) + \sum_q \lambda_q^{(d)}\left(2A_{3,q}^P + A_{15,q}^P\right) \\
&= -I_0 + 2I_2, \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}(B_d \to \pi^+\pi^-) &= -\lambda_u^{(d)}(C_3^T + C_6^T + 3C_{15}^T) - \sum_q \lambda_q^{(d)}(C_{3,q}^P + C_{6,q}^P + 3C_{15,q}^P) \\
&\quad -\lambda_u^{(d)}\left(2A_3^T + A_{15}^T\right) - \sum_q \lambda_q^{(d)}\left(2A_{3,q}^P + A_{15,q}^P\right) \\
&= I_0 + I_2. \qquad (3)
\end{aligned}
$$

Here, $\lambda_q^{(q')} = V_{qb}^\star V_{qq'}$ ($q = u, c, t$ and $q' = d, s$) and C_q^P, A_q^P denote the penguin amplitudes due to q running in the loop. [4] I_2 and I_0 are the amplitudes for $B \to \pi\pi(I = 2)$ and $(I = 0)$ respectively or in other words the $\Delta I = 3/2$ and $\Delta I = 1/2$ amplitudes.

Using the unitarity of the CKM matrix, *i.e.*, $\lambda_t^{(d)} = -\lambda_u^{(d)} - \lambda_c^{(d)}$, we get

$$
\lambda_u^{(d)} C_i^T + \sum_q \lambda_q^{(d)} C_{i,q}^P = \lambda_u^{(d)} \tilde{C}_i^T - \lambda_c^{(d)} C_i^P \qquad (4)
$$

where $\tilde{C}_i^T = C_i^T - C_{i,t}^P + C_{i,u}^P$ and $C_i^P = C_{i,t}^P - C_{i,c}^P$. A similar notation is used for $\tilde{A}_i^T$ and A_i^P.

In the $B^+ \to \pi^+\pi^0$ decay, which contains only the $\Delta I = 3/2$ amplitude, there is no contribution to $C_{15,q}^P$ from the strong penguin diagrams since these diagrams are $\Delta I = 1/2$. Neubert and Rosner [9] showed that $C_{15,q}^P = C_{15}^T\, 3/2\, \kappa_q$, where $\kappa_q = (c_{9,q} + c_{10,q})/(c_1 + c_2)$ is the ratio of Wilson coefficients (WC's) of the EWP operators (with quark q running in the loop) and the tree level operators in the effective Hamiltonian. We expect $c_{(9,10),t} \gg c_{(9,10),(u,c)}$ since the top quark EWP diagram with Z exchange is enhanced by m_t^2/m_Z^2 and so henceforth we neglect $\kappa_{u,c}$ and denote κ_t by κ. So, we get

$$
\tilde{C}_{15}^T \approx C_{15}^T \left(1 - \frac{3}{2}\kappa\right), \qquad (5)
$$

$$
C_{15}^P \approx C_{15}^T \frac{3}{2}\kappa \qquad (6)
$$

[4] Tree level operators with the flavor structure $c\bar{c}\,\bar{b}d$ can also contribute through rescattering from charm intermediate states. This rescattering generates a charm-quark penguin topology with the amplitude being proportional to $\lambda_c^{(d)}$ and so can be included as part of $C_{i,c}^P$.

Using Eqns.(4), (5) and (6), we can rewrite $-3\ I_2 = 8\ (\lambda_u^{(d)} C_{15}^T + \sum_q \lambda_q^{(d)} C_{15,q}^P)$ $= 8\ \left(\lambda_u^{(d)} \tilde{C}_{15}^T - \lambda_c^{(d)} C_{15}^P\right)$ as

$$-3\ I_2 \ \approx\ 8\ \tilde{C}_{15}^T \left(\lambda_u^{(d)} - \frac{\frac{3}{2}\kappa}{1-\frac{3}{2}\kappa}\lambda_c^{(d)}\right). \tag{7}$$

Since $3/2\ \kappa \sim 2\%$ and $|\lambda_u^{(d)}| \sim |\lambda_c^{(d)}|$, we neglect the second term (*i.e.*, the EWP contribution) in the right hand side of Eqn.(7) for now and we assume $|\tilde{C}_{15}^T| \approx |C_{15}^T|$. We will show later how to include it. Then, using the Wolfenstein parametrization in which $\lambda_u^{(d)} = |\lambda_u^{(d)}| e^{i\gamma}$ and $\lambda_c^{(d)} = |\lambda_c^{(d)}|$ (and similarly for d replaced by s), we get

$$-3\ I_2 \approx |\lambda_u^{(d)}| e^{i\gamma}\ 8\ |\tilde{C}_{15}^T| \approx |\lambda_u^{(d)}| e^{i\gamma}\ 8\ |C_{15}^T| \tag{8}$$

so that $|C_{15}^T|$ can be obtained directly from the $B^+ \to \pi^+\pi^0$ decay rate. We have chosen a phase convention such that the strong phase of C_{15}^T is zero.

From Eqns.(2) and (3), we get

$$\begin{aligned} I_0 &= -\lambda_u^{(d)} \left(\tilde{C}_3^T + \tilde{C}_6^T + \frac{1}{3}\tilde{C}_{15}^T\right) + \lambda_c^{(d)} (C_3^P + C_6^P + \frac{1}{3} C_{15}^P) \\ &\quad -\lambda_u^{(d)} \left(2\ \tilde{A}_3^T + \tilde{A}_{15}^T\right) + \lambda_c^{(d)} \left(2A_3^P + A_{15}^P\right) \\ &\equiv e^{i\phi_{\tilde{T}}}\ |\lambda_u^{(d)}|\ e^{i\gamma}\ \tilde{T} - |\lambda_c^{(d)}|\ e^{i\phi_P}\ P. \end{aligned} \tag{9}$$

The five quantities: $|C_{15}^T|$, $\tilde{T}$, P, $\phi_{\tilde{T}}$ and ϕ_P (where the ϕ's are the CP conserving strong phases) can thus be determined as functions of γ from the measurements of the five rates: $B^+ \to \pi^+\pi^0$, $B_d \to \pi^+\pi^-$, $B_d \to \pi^0\pi^0$ and the CP-conjugates of the B_d decays.

Explicitly, rotating the CP-conjugate amplitudes by $e^{i2\gamma}$ (and denoting them by "bars"), we get the triangle formed by $B^+ \to \pi^+\pi^0$, $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ (Eqns.(1), (2) and (3)):

$$-\sqrt{2}\mathcal{A}(B^+ \to \pi^+\pi^0) + \sqrt{2}\mathcal{A}(B_d \to \pi^0\pi^0) + \mathcal{A}(B_d \to \pi^+\pi^-) = 0 \tag{10}$$

and the one formed by the CP-conjugate decays. These are shown in Fig.1 (from Eqn.(8), $I_2 = \bar{I}_2$).

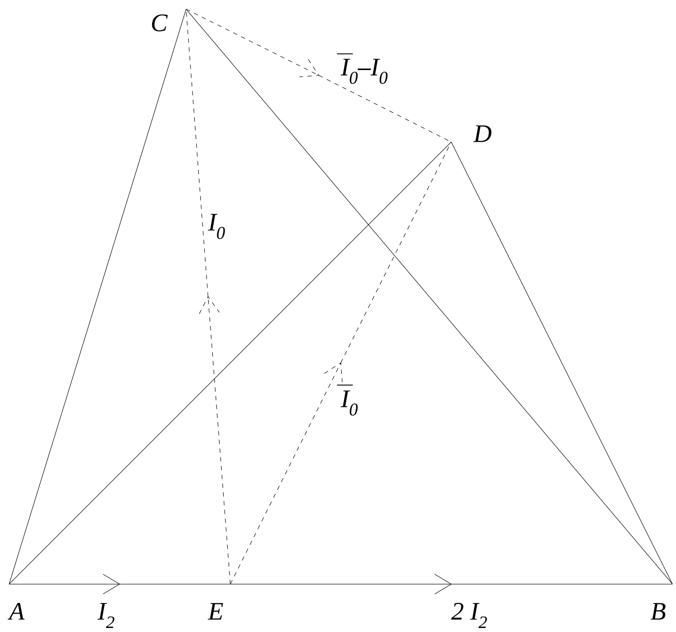

Figure 1: The triangles formed by the $B \to \pi\pi$ amplitudes: $AB = |\sqrt{2}\mathcal{A}\left(B^+ \to \pi^+\pi^0\right)|$, $AC = |\mathcal{A}\left(B_d \to \pi^+\pi^-\right)|$, $BC = |\sqrt{2}\mathcal{A}\left(B_d \to \pi^0\pi^0\right)|$, $AD = |\mathcal{A}\left(\bar{B}_d \to \pi^+\pi^-\right)|$ and $BD = |\sqrt{2}\mathcal{A}\left(\bar{B}_d \to \pi^0\pi^0\right)|$. In the phase convention where the strong phase of C_{15}^T is zero, the angle between I_2 and the real axis (not shown) is $\pi + \gamma$ (see Eqn.(8)).

From Eqn.(9), we get

$$\bar{I}_0 - I_0 = -|\lambda_c^{(d)}|e^{i\phi_P} P \left(e^{i2\gamma} - 1\right). \tag{11}$$

Thus, the length of $\bar{I}_0 - I_0$ (obtained from Fig.1) gives P as a function of γ. The angle between $\bar{I}_0 - I_0$ and I_2 is $\phi_P + \pi/2$ (see Eqns.(8) and (11)) so that the orientation of $\bar{I}_0 - I_0$ (obtained from Fig.1) gives ϕ_P independent of γ. [5]

Similarly,

$$\bar{I}_0 e^{-i2\gamma} - I_0 = |\lambda_u^{(d)}|e^{i\phi_{\tilde{T}}}\tilde{T}\left(e^{-i\gamma} - e^{i\gamma}\right) \tag{12}$$

gives $\phi_{\tilde{T}}$ and $\tilde{T}$ (or knowing I_0, P and ϕ_P gives $\phi_{\tilde{T}}$ and $\tilde{T}$ using Eqn.(9)).

There is a discrete ambiguity in this procedure since in Fig.1, the vertices C and D could be on opposite sides of I_2. This has been discussed in the literature [10] [11].

[5] This was also discussed in [10].

All the analysis up to now actually relies only on flavor $SU(2)$ symmetry.

2.1 Case 1

The $B_d \to K\pi$ and $B_s \to \pi\pi$ amplitudes are given by [8]

$$\begin{aligned}
-\mathcal{A}(B_d \to K^+\pi^-) &= \lambda_u^{(s)}(C_3^T + C_6^T + 3C_{15}^T) + \sum_q \lambda_q^{(s)} \left(C_{3,q}^P + C_{6,q}^P + 3C_{15,q}^P\right) \\
&\quad -\lambda_u^{(s)} A_{15}^T - \sum_q \lambda_q^{(s)} A_{15,q}^P, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\sqrt{2}\,\mathcal{A}(B_d \to \pi^0 K^0) &= \lambda_u^{(s)}(C_3^T + C_6^T - 5C_{15}^T) + \sum_q \lambda_q^{(s)} \left(C_{3,q}^P + C_{6,q}^P - 5C_{15,q}^P\right) \\
&\quad -\lambda_u^{(s)} A_{15}^T - \sum_q \lambda_q^{(s)} A_{15,q}^P, \qquad (14)
\end{aligned}$$

$$\begin{aligned}
2\,\mathcal{A}\left(B_s \to \pi^+\pi^-\right) &= \mathcal{A}\left(B_s \to \pi^0\pi^0\right) \\
&= -\lambda_u^{(s)} \left(4\,A_3^T + 4\,A_{15}^T\right) \\
&\quad -\sum_q \lambda_q^{(s)} \left(4\,A_{3,q}^P + 4\,A_{15,q}^P\right). \qquad (15)
\end{aligned}$$

From Eqns.(13) and (15) and using the notation of Eqn.(4) and rearranging we get

$$\begin{aligned}
\mathcal{A}(B_d \to K^+\pi^-) + \mathcal{A}\left(B_s \to \pi^+\pi^-\right) &= -\lambda_u^{(s)} \left(\tilde{C}_3^T + \tilde{C}_6^T + \frac{1}{3}\tilde{C}_{15}^T\right) \\
&\quad +\lambda_c^{(s)}(C_3^P + C_6^P + \frac{1}{3}C_{15}^P) \\
&\quad -\lambda_u^{(s)} \left(2\,\tilde{A}_3^T + \tilde{A}_{15}^T\right) + \lambda_c^{(s)} \left(2A_3^P + A_{15}^P\right) \\
&\quad -\frac{8}{3}\left(\lambda_u^{(s)}\tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P\right). \qquad (16)
\end{aligned}$$

Using Eqns.(6) and (9) and assuming $C_{15}^T \approx \tilde{C}_{15}^T$, we get

$$\begin{aligned}
\mathcal{A}(B_d \to K^+\pi^-) + \mathcal{A}\left(B_s \to \pi^+\pi^-\right) &= e^{i\phi_{\tilde{T}}}|\lambda_u^{(s)}|e^{i\gamma}\tilde{T} - |\lambda_c^{(s)}|e^{i\phi_P}P \\
&\quad -\frac{8}{3}C_{15}^T\left(\lambda_u^{(s)} - \frac{3}{2}\,\kappa\,\lambda_c^{(s)}\right), \qquad (17)
\end{aligned}$$

i.e., from Eqns.(3) and (16), we see that the combination of the amplitudes $\mathcal{A}(B_d \to K^+\pi^-) + \mathcal{A}\,(B_s \to \pi^+\pi^-)$ can be obtained from the amplitude for $B_d \to \pi^+\pi^-$ by scaling the tree level contribution in the latter by $|\lambda_u^{(s)}|/|\lambda_u^{(d)}|$ and the penguin contribution by $|\lambda_c^{(s)}|/|\lambda_c^{(d)}|$. In particular the EWP contribution ($\propto \lambda_c^{(s)}$) is important in the last line of Eqn.(17) since, due to the CKM factors, it is comparable to the tree level contribution (unlike in the $B_d \to \pi^+\pi^-$ decay; see Eqn.(7)).

Similarly, from Eqns.(2), (14) and (15), we see that the combination of the amplitudes $\sqrt{2}\,\mathcal{A}(B_d \to K^0\pi^0) - \mathcal{A}\,(B_s \to \pi^-\pi^+)$ can be obtained from the amplitude $\sqrt{2}\,\mathcal{A}\,(B_d \to \pi^0\pi^0)$ by scaling the latter by CKM factors. This gives

$$\begin{aligned}\sqrt{2}\,\mathcal{A}(B_d \to K^0\pi^0) - \mathcal{A}\left(B_s \to \pi^+\pi^-\right) &= -e^{i\phi_{\tilde{T}}}|\lambda_u^{(s)}|e^{i\gamma}\tilde{T} + |\lambda_c^{(s)}|e^{i\phi_P}P \\ &\quad -\frac{16}{3}C_{15}^T\left(\lambda_u^{(s)} - \frac{3}{2}\,\kappa\,\lambda_c^{(s)}\right). \end{aligned} \tag{18}$$

We emphasize again that in obtaining the last lines of Eqns.(17) and (18), it is crucial that we use the Neubert-Rosner result (Eqn.(6)), *i.e.,* that κ is calculable. If $\mathcal{A}\,(B_s \to \pi^+\pi^-) \equiv a'\ e^{i\phi_a'}$, we can determine a' from the decay rate $B_s \to \pi^+\pi^-$. Then, measuring the decay rate $B_d \to \pi^-K^+$ gives ϕ_a' as a function of γ (using Eqn.(17)) (ϕ_P, $\phi_{\tilde{T}}$, P and $\tilde{T}$ are already known as functions of γ). Knowing a' and ϕ_a', we have a prediction for the decay rate $B_d \to K^0\pi^0$ (Eqn.(18)) and then γ can be determined by measuring this decay. [6] Thus, we can determine γ, including rescattering effects, by measuring the 8 decay modes: $B^+ \to \pi^+\pi^0$, B_d and $\bar{B}_d \to \pi^+\pi^-, \pi^0\pi^0$, $B_d \to \pi^-K^+$, $B_d \to K^0\pi^0$ and $B_s \to \pi\pi$ (any one) (or CP-conjugates of the last three modes).

From Eqn.(15), we see that the decay mode $B_s \to \pi\pi$ has only an annihilation contribution. If (either from experimental measurement of the $B_s \to \pi\pi$ rate (or a limit on the rate) or a theoretical estimate including

[6] In this part of the technique, we introduce two additional discrete ambiguities – one in determining ϕ_a' and another in the final determination of γ from the $B_d \to K^0\pi^0$ rate.

rescattering) the annihilation amplitude $\sim A_3 + A_{15}$ does turn out to be small (smaller than, say, the experimental error in the measurement of the (magnitude) of $\mathcal{A}\left(B_d \to \pi K\right)$) then, a decay rate $B_d \to \pi K$ can be predicted (as a function of γ) by simply scaling the corresponding $B_d \to \pi\pi$ amplitude by CKM factors. Thus, in this case, 6 decay modes: $B^+ \to \pi^+\pi^0$, B_d (and $\bar{B}_d$) $\to \pi^+\pi^-, \pi^0\pi^0$ and any *one* $B_d \to \pi K$ are sufficient to determine γ.

As mentioned in the introduction, we require tagging to measure the $B_s \to \pi\pi$ decay mode (*i.e.*, the CP-averaged rate is not sufficient in the above method). If this tagged decay rate is hard to measure whereas the CP-averaged rate can be measured, we can proceed as follows. Writing $\mathcal{A}\left(\bar{B}_s \to \pi^+\pi^-\right) \equiv \bar{a}' e^{\bar{\phi}'_a}$, the CP-averaged rate is $1/2\left(a'^{\,2} + \bar{a}'^{\,2}\right)$. Measuring the $B_d \to \pi K$ decay rates, from Eqns.(17) and (18), we can determine a' (and ϕ'_a) as a function of γ. Similarly, the $\bar{B}_d \to \pi K$ decay rates will give $\bar{a}'$. Thus, the CP-averaged $B_s \to \pi\pi$ rate can be predicted as a function of γ and its measurement gives γ.

2.2 Case 2

The expression for the $B_s \to K^+K^-$ decay amplitude in terms of the $SU(3)$ invariant amplitudes is identical to that for $B_d \to \pi^-\pi^+$, *including* annihilation contributions (unlike the decay mode $B_d \to \pi^-K^+$), modulo CKM factors, *i.e.,* up to $\lambda_q^{(d)} \to \lambda_q^{(s)}$ [8]. Thus, we also have a prediction (as a function of γ) for this decay rate and also the rate for its CP-conjugate process including *all* rescattering effects. So, the measurement of this CP-averaged decay rate can be used to determine γ.

3 Cases 3 and 4

Using the same technique as in section 2, we can *predict* the rate for a $B_d \to \pi\pi$ (or a $B_s \to \pi K$) decay as a function of γ *given* the decay rates for $B_d \to \pi K$. The difference is that in the triangle construction to determine

the tree level and penguin contributions to the $B_d \to \pi K$ amplitudes, we have to take into account the EWP contributions to the $B_d \to \pi K$ amplitudes (the EWP amplitudes were neglected in the $B \to \pi\pi$ triangles in section 2; see Eqns.(7) and (8)). [7] So, we discuss the application of the technique again.

The decay amplitudes for $B_d \to \pi K$ (Eqns.(13) and (14)) can be written as

$$\sqrt{2}\,\mathcal{A}(B_d \to \pi^0 K^0) = -I_{1/2} + 2I_{3/2}, \tag{19}$$

$$\mathcal{A}(B_d \to \pi^- K^+) = I_{1/2} + I_{3/2}, \tag{20}$$

where $I_{1/2}$ and $I_{3/2}$ are the amplitudes for B_d decay to $\pi\ K$ ($I = 1/2$) and ($I = 3/2$) respectively. Then,

$$\begin{aligned} 3I_{3/2} &= \sqrt{2}\mathcal{A}(B_d \to \pi^0 K^0) + \mathcal{A}(B_d \to \pi^- K^+) \\ &= -\lambda_u^{(s)} 8\tilde{C}_{15}^T + 8\lambda_c^{(s)} C_{15}^P \\ &= -8C_{15}^T \left(\lambda_u^{(s)} - \frac{3}{2}\kappa\lambda_c^{(s)}\right) \\ &= -8\,|C_{15}^T||\lambda_u^{(s)}|\left(e^{i\gamma} + \delta_{EW}\right), \end{aligned} \tag{21}$$

using Eqn.(6) and $|\tilde{C}_{15}^T| \approx |C_{15}^T|$ (*i.e.,* neglecting the EWP contribution in the $B^+ \to \pi^+\pi^0$ decay). δ_{EW} is given by $-|\lambda_c^{(s)}|/|\lambda_u^{(s)}|\ 3/2\ \kappa \sim O(1)$, *i.e.,* as mentioned earlier, the EWP contribution *is* important for $B_d \to \pi K$ decays. $|C_{15}^T|$ can be obtained from the $B^+ \to \pi^+\pi^0$ decay rate as before.

$I_{1/2}$ is given by (in analogy to I_0 of section 2)

$$\begin{aligned} I_{1/2} &= -\lambda_u^{(s)}\left(\tilde{C}_3^T + \tilde{C}_6^T + \frac{1}{3}\tilde{C}_{15}^T\right) + \lambda_c^{(s)}(C_3^P + C_6^P + \frac{1}{3}C_{15}^P) \\ &\quad +\lambda_u^{(s)}\tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \\ &\equiv e^{i\phi'_{\tilde{T}}}|\lambda_u^{(s)}|e^{i\gamma}\tilde{T}' - |\lambda_c^{(s)}|e^{i\phi'_P}P'. \end{aligned} \tag{22}$$

As in section 2, the four quantities: $\tilde{T}'$, P', $\phi'_{\tilde{T}}$ and ϕ'_P can thus be determined as functions of γ from the measurements of the four decay rates:

[7] Of course, in section 2, to make a *prediction* for $\Delta S = 1$ decay modes, we did have to include the EWP contribution to the $\Delta S = 1$ decay amplitudes.

$B_d \to \pi^- K^+$, $B_d \to \pi^0 K^0$ and their CP-conjugates. [8]

Due to the EWP contribution (see Eqn.(21)), the triangle construction is a bit different in this case as shown below.

As before, we multiply the CP-conjugate amplitudes by $e^{i2\gamma}$ to get the "barred" amplitudes. In this case (unlike the case for I_2 in section 2) there is an angle between $I_{3/2}$ and $\bar{I}_{3/2}$ denoted by $2\tilde{\gamma}$ and their magnitudes are functions of γ (see Eqn.(21)):

$$|I_{3/2}| = |\bar{I}_{3/2}| = \frac{8}{3}|C_{15}^T||\lambda_u^{(s)}|\sqrt{(1+\delta_{EW}^2 + 2\delta_{EW}\cos\gamma)}, \tag{23}$$

$$\tan\tilde{\gamma} = \frac{\delta_{EW}\sin\gamma}{1+\delta_{EW}\cos\gamma}. \tag{24}$$

Given γ, we can thus construct the triangles of Eqn.(21) and its CP-conjugate (see Fig.2). [9] As in section 2, knowing the magnitudes and orientations of $I_{1/2}$ and $\bar{I}_{1/2}$ from Fig.2, we can determine $\tilde{T}'$, P', $\phi'_{\tilde{T}}$ and ϕ'_P as functions of γ, using equations similar to Eqns.(11) and (12).

This construction also shows how to include the EWP contributions to the $B^+ \to \pi^+\pi^0$ decay in section 2 as follows.

Once EWP's are included, as for the case of $I_{3/2}$ and $\bar{I}_{3/2}$, there is an angle $2\,\tilde{\gamma}'$ between I_2 and $\bar{I}_2$ and the magnitude of $\tilde{C}_{15}^T$ will depend on γ (see Eqn.(7)):

$$\begin{aligned} 3\,|I_2| = 3\,|\bar{I}_2| &= \left|\sqrt{2}\mathcal{A}\left(B^+ \to \pi^+\pi^0\right)\right| \\ &= 8|\tilde{C}_{15}^T||\lambda_u^{(d)}|\sqrt{(1+\delta_{EW}'^2 + 2\delta'_{EW}\cos\gamma)}, \end{aligned} \tag{25}$$

$$\tan\tilde{\gamma}' = \frac{\delta'_{EW}\sin\gamma}{1+\delta'_{EW}\cos\gamma}, \tag{26}$$

where δ'_{EW} is given by $-|\lambda_c^{(d)}|/|\lambda_u^{(d)}|\;3/2\;\kappa\;/\;(1-3/2\;\kappa) \sim O(\text{few}\;\%)$.

[8] A simlilar analysis can be done with the $B^+ \to \pi K$ decay amplitudes which can be written in terms of $I_{3/2}$ (the same as for $B_d \to \pi K$ decays) and $I'_{1/2}$ which is a different combination of the $SU(3)$ invariant amplitudes than $I_{1/2}$. Thus, the $B^+ \to \pi K$ decay rates are not useful as far as predicting the $B_d \to \pi\pi$ (case 3) (or $B_s \to \pi K$ in case 4) decay rates is concerned.

[9] As in section 2, there is a discrete ambiguity in the orientation of the triangles.

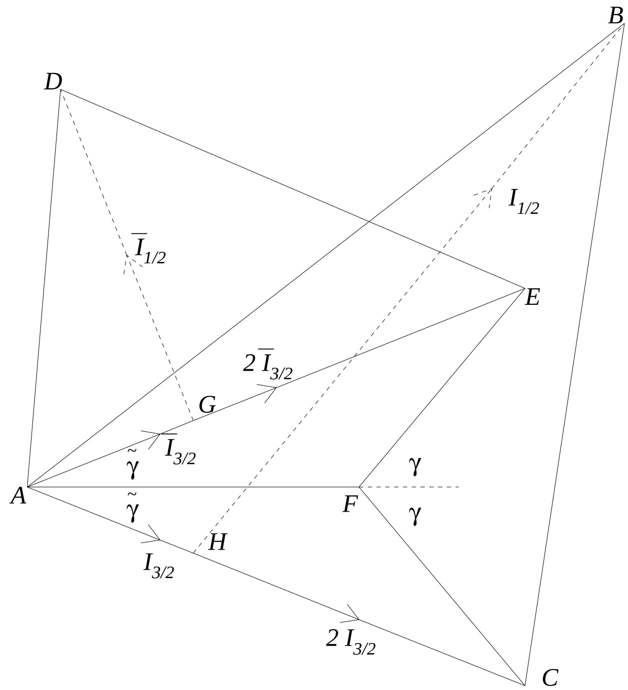

Figure 2: The triangles formed by the $B_d \to \pi K$ amplitudes: $AB = |\mathcal{A}\left(B_d \to K^+\pi^-\right)|$, $BC = |\sqrt{2}\mathcal{A}\left(B_d \to \pi^0 K^0\right)|$, $AD = |\mathcal{A}\left(\bar{B}_d \to K^-\pi^+\right)|$, $DE = |\sqrt{2}\mathcal{A}\left(\bar{B}_d \to \pi^0 \bar{K}^0\right)|$. $AF = |\sqrt{2}\ \mathcal{A}\left(B^+ \to \pi^+\pi^0\right)|\ |\lambda_u^{(s)}|/|\lambda_u^{(d)}|$ and $FC = FE = AF\ \delta_{EW}$ (see Eqn.(21)). As in Fig.1, in the phase convention where the strong phase of C_{15}^T is zero, the angle between AF and the real axis is $\pi + \gamma$.

3.1 Case 3

The $B_d \to K^+K^-$ amplitude is given by [8]:

$$\begin{aligned} \mathcal{A}\left(B_d \to K^+K^-\right) &= -\lambda_u^{(d)}\left(2A_3^T + 2A_{15}^T\right) - \sum_q \lambda_q^{(d)}\left(2A_{3,q}^P + 2A_{15,q}^P\right) \\ &\equiv a e^{i\phi_a}. \end{aligned} \tag{27}$$

From Eqns.(2), (3), (13), (14) and (27), we can see that $\sqrt{2}\mathcal{A}\left(B_d \to \pi^0\pi^0\right) + \mathcal{A}\left(B_d \to K^-K^+\right)$ can be obtained from $\sqrt{2}\mathcal{A}\left(B_d \to K^0\pi^0\right)$ and $\mathcal{A}\left(B_d \to \pi^+\pi^-\right) - \mathcal{A}\left(B_d \to K^-K^+\right)$ can be obtained from $\mathcal{A}\left(B_d \to \pi^-K^+\right)$ by scaling the $\Delta S = 1$ amplitudes by appropriate CKM factors, *i.e.,* in this case, the decay mode $B_d \to K^-K^+$ plays the role of the decay mode $B_s \to \pi\pi$ of case 1 (compare Eqns.(15) and (27)). Thus, as in case 1, we can determine γ, including *all* rescattering effects, by measuring the 8 decay modes: $B^+ \to \pi^+\pi^0$, B_d and $\bar{B}_d \to \pi K$ (all), $B_d \to K^+K^-$, $B_d \to \pi^0\pi^0$ and

$B_d \to \pi^-\pi^+$ (or CP-conjugates of the last three modes). If the annihilation amplitudes are small, as in case 1, we can determine γ by measuring any *one* $B_d \to \pi\pi$ decay mode, in addition to the $B^+ \to \pi^+\pi^0$, B_d (and $\bar{B}_d$) $\to \pi K$ decay modes. As in case 1, if we measure the CP-conjugate $B_d \to \pi\pi$ rates as well, then a CP-averaged measurement of the decay rate $B_d \to K^+K^-$ suffices.

3.2 Case 4

The expressions for the decay amplitudes for $B_s \to \pi^+K^-$ and $B_s \to \pi^0\bar{K}^0$ in terms of the $SU(3)$ invariant amplitudes are identical to those for $B_d \to \pi^-K^+$ and $B_d \to \pi^0K^0$, respectively, *including* annihilation contributions (unlike the case of $B_d \to \pi\pi$ and $B_d \to \pi K$ decays), modulo the CKM factors [8]. Thus, the same method predicts the rates for the $B_s \to \pi K$ decays and the CP-conjugate processes. It suffices to use the $B_s \to K^-\pi^+$ decay (or its CP-conjugate) which is a "self tagging" mode or the CP-averaged decay rate for $B_s \to \pi^0\bar{K}^0$. Thus, no external tagging is required.

4 Discussions

The analysis of the above three cases is strictly valid only in the flavor $SU(3)$ limit. In the tree level amplitudes, *i.e.,* C_{15}^T and the $\tilde{T}$'s, the corrections due to $SU(3)$ breaking can be taken into account more reliably in the factorization approximation and are expected to be given by f_K/f_π (see, for example, Gronau *et al.* in [1] [10]). For example, in Fig.2 the lengths of AF, FE and FC will have to be multiplied by f_K/f_π. However, since the strong penguin amplitudes include $(V-A)(V+A)$ type operators, the (factorizable) corrections due to $SU(3)$ breaking there are less certain, but the corrections are still less than $\sim O(30\%)$. This is especially relevant for the cases 1 and 2 where the penguin contribution dominates in the $B_d \to \pi K$, $B_s \to K^+K^-$ decays and we are *predicting* this contribution from the $B_d \to \pi\pi$ decays

using $SU(3)$ symmetry. In the cases 3 and 4, we use $SU(3)$ symmetry to predict the penguin contribution in a $B_d \to \pi\pi$ (or a $B_s \to \pi K$) decay from the $B_d \to \pi K$ decays, but now the *tree level* contributions dominate the $B_d \to \pi\pi$ decay rate and so the uncertainty due to the $SU(3)$ breaking in the penguin amplitudes is less important.

We have a prediction for more than one rate in some of the cases. For example, in case 4, we can predict both $B_s \to \pi^+ K^-$ and its CP-conjugate decay rate or in case 1, neglecting annihilation, we can predict the decay rates $B_d \to \pi^- K^+$ and $\pi^0 K^0$. So, we can treat the $SU(3)$ breaking in the penguin amplitudes as an unknown and determine it (in addition to γ) from the measurement of *seven* decay rates.

We have also assumed that the $SU(3)$ breaking in the strong phases is small. A possible justification is that at the energies of the final state particles $\sim m_b/2$, the phase shifts are not expected to be sensitive to the $SU(3)$ breaking given by, say, $m_K - m_\pi$ (which is much smaller than the final state momenta). However, it is hard to quantify this effect.

If we measure *all* the $B \to \pi\pi$ and $B_d \to \pi K$ decay rates, then we can compute the tree level parts of the amplitudes, both $\tilde{T}$ and $\tilde{T}'$ (see Eqns.(9) and (22)), as functions of γ as discussed in sections 2 and 3. If the annihilation amplitudes are small, then we have $\tilde{T} = \tilde{T}'$ (in the $SU(3)$ limit) since the decay amplitudes $B_d \to \pi\pi$ and $B_d \to \pi K$ are the same up to CKM factors. To include the $SU(3)$ breaking in this analysis, we use the modified relations $C_{15}^T\ (\Delta S = 1) = f_K/f_\pi\ C_{15}^T\ (\Delta S = 0)$ and $\tilde{T}' = f_K/f_\pi\ \tilde{T}$. This can be used to determine γ, including $SU(3)$ breaking (without having to to deal with $SU(3)$ breaking in the penguin amplitudes and in the strong phases).

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